Econ 802

Final Exam

Greg Dow December 16, 2020

All questions have equal weight. If you are not making any progress on a question, it is a good idea to move on to something else and come back to it later.

1. A clever undergraduate student makes the following statements. In each case, construct an <u>exception</u> to the student's claim. Explain using graphs.

(a) "If a firm has a positive marginal product for every input, then it uses a positive amount of every input."

(b) "If a consumer minimizes expenditure, then her Hicksian demand curves must be downward sloping."

(c) "If a market has free entry and exit in the long run, and all firms are price takers, then there is a price level where supply equals demand."

2. A production function $f(x)$ with $x = (x_1 \,.\,.\, x_n) \geq 0$ is differentiable, strictly quasi-concave, and has positive marginal products. The input prices are $w = (w_1 \,.\,.\, w_n) > 0$. Assume all optimization problems have interior solutions.

(a) Define the function $g(w, e) \equiv \max f(x)$ subject to $wx \leq e$ where $e > 0$ is a scalar. Describe the mathematical properties of $g(w, e)$. Carefully justify your answers.

(b) Define the cost function $c(w, y) \equiv \min wx$ subject to $f(x) \geq y$. If you happen to know $c(w, y)$, can you find $g(w, e)$? If you know $g(w, e)$, can you find $c(w, y)$? Give a mathematical argument and explain using a graph for the case $n = 2$.

(c) Suppose you know $g(w, e)$. Describe a mathematical method you could use to find the production function $f(x)$. Carefully justify each step in your argument and explain using a graph for the case $n = 2$.

3. Kamala cares about $x \geq 0$ and $y \geq 0$. Her utility function is $u = 1 - e^{-rx} + y$ where $r > 0$. The price of good x is p where $0 < p \leq r$. The price of good y is always one. Kamala is endowed with w units of the y good but none of the x good.

(a) Find the Marshallian and Hicksian demand functions for the x good. Show that in each case your solution satisfies a sufficient second order condition.

(b) Solve for the indirect utility function $v(p, w)$. If there are many small Kamalas $i = 1 \ldots n$ with different preference parameters r_i and different endowments w_i, would it make sense to treat the aggregate market demand $X(p) = \sum x_i(p)$ for the x good as if it came from one big Kamala? Justify your answer.

(c) The x good is not currently available but the government could sell a fixed total supply $X^0 > 0$ at an equilibrium price p^0. It would cost the government $F > 0$ to produce X^0 where F is measured in terms of the y good. The government will do this if and only if the area under the market demand curve $X(p)$ up to the amount X^0 exceeds F. Does this plan make economic sense? Explain using a graph.

4. Firms in the widget industry have the production function $y = (x_1x_2)^{1/2}$ where $x_1 \geq 0$ and $x_2 \geq 0$. The input prices are always $w_1 = w_2 = 1$. The price of output is $p > 0$. Both in the short run and in the long run, the number of firms is fixed at n.

(a) In the short run, $x_2 > 0$ is fixed for each firm. Find the supply function $y(p)$ for an individual firm and show it on a graph. Does the firm ever shut down in the short run? Use your graph to explain why or why not.

(b) Let $x_{j2} > 0$ be the level of the fixed input for firm $j = 1 \ldots n$. Solve for the short run market supply function $S(p)$. Then assume the market demand function is $D(p) = Ap^{-b}$ where $A > 0$ and $b > 0$. Solve for the short run equilibrium price p^* as a function of the exogenous parameters and show your solution on a graph.

(c) In the long run, the firms $j = 1 \ldots n$ can choose the levels of both inputs. Using the same demand function as in (b), solve for the equilibrium price p^* and show your solution on a graph. Is it possible for the price to be lower in the short run than in the long run? Carefully explain why or why not.

5. There are consumers $i = A, B$ and goods $j = 1, 2$. A's consumption bundle is $(x_{A1}, x_{A2}) \geq 0$ and B's bundle is $(x_{B1}, x_{B2}) \geq 0$. A has the utility function $u_A = \alpha \ln x_{A1} + (1-\alpha) \ln x_{A2}$ and B has the utility function $u_B = \beta \ln x_{B1} + (1-\beta) \ln x_{B2}$ where $0 < \alpha < 1$ and $0 < \beta < 1$.

(a) A's endowment vector is $(w_{A1}, w_{A2}) > 0$ and B's endowment vector is $(w_{B1}, w_{B2}) > 0$. Solve for the Walrasian equilibrium price ratio p_2/p_1. Explain your reasoning.

(b) Assume instead there is a social planner who maximizes $au_A + bu_B$ where $a > 0$, $b > 0$, and $a + b = 1$. The total supply of good 1 is W_1 and the total supply of good 2 is W_2. Solve for the ratio of the Lagrange multipliers q_2/q_1 associated with the two physical feasibility constraints. Explain your reasoning.

(c) Taking the weights (a, b) as given, find individual endowment vectors (w_{A1}, w_{A2}) and (w_{B1}, w_{B2}) such that $p_2/p_1 = q_2/q_1$. Give a detailed verbal interpretation.